MATH 5051 Algebra

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This course will cover the content below:
(1) Categories and Functors;
(2) Group and categories;
(3) Tensor products;
(4) Galois Theory and etale algebras over a field;
(5) Semisimple algebras

1 Category and Functors

In this course, we normally use $\mathcal{C}$ to represent a category.

Definition 1 *A category $\mathcal{C}$ consists of objects, morphisms, domains and codomains, where Ob($\mathcal{C}$) and Mor($\mathcal{C}$) are classes, (when they are sets we call it a small category).*
(1)Domain and Codomain assigin each $f \in$ Mor($\mathcal{C}$) an object respectively , denoted as dom(f) and codom(f).
(2)For any A,B are objects, Hom(A,B) = $\{f \in$ Mor($\mathcal{C}$), dom(f)=A, codom(f) = B
(3)$For f, g\in$ Mor($\mathcal{C}$), the composition f and g ($f \circ g$) can be defined if dom(f)=codom(g). With all these datas, they must satifies:
(Composition Law) $f \circ (g \circ h) = (f \circ g) \circ h$
(Existence of identity morphism) For any object A, there exists $Id_A \in$ Hom(A,A), s.t., $f \circ Id_A = Id_A \circ f = f$

Examples
(1) The category of sets, rings groups, etc.
(2) Category of vector spaces.
(3) Category of manifolds

Definition 2 *$\mathcal{C}$ is a category. $f \in$ Hom(C,D), we say f is an isomorphiam if there exists $g \in$ Hom(D,C), s.t.:*
$g \circ f = Id_c$, $f \circ g = Id_d$.

Definition 3 *$\mathcal{C}$ is a category. X is an initial object if for any $Y \in Ob(\mathcal{C})$, the carnality of Hom(X,Y)=1;*
call X is a final object if for any $Y \in Ob(\mathcal{C})$, the carnality of Hom(Y,X)=1;

Examples

(1)**Sets**, the initial object is an empty set.
(2)**Groups**,the initial object is the trivial group.